## THERMAL PROBLEMS IN MODELING OF WELDING BY A FIXED FUSING ELECTRODE

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A theoretical description of the thermal processes occurring in the electrode and the base metal is given. Useful mathematical models are suggested for calculating the dynamics of temperature fields, and the reasons for defects in the weld are discussed.

Introduction. A group of researchers at Moscow Power Engineering Institute, directed by Prof. I. V. Zuev, has developed a new method of electric arc welding to be used for large-sized pieces [1, 2]. A fixed plate electrode is located between the pieces to be welded and is separated from them by insulating pads. In Fig. 1 part of the assembly, including one of the pieces, an insulator, and the electrode, is shown. The complete assembly contains the other insulator and the second piece to be welded, located symmetrically about the electrode. The arc is ignited between the electrode and the base metal. Oscillating rapidly in the 0Y direction, the are produces a channel with the melting electrode as the upper wall, the base metal of the pieces as side walls, and the weld metal as the lower wall. Transfer of melt from the electrode to the weld displaces the channel upward in the 0Z direction. In this process the insulator burns out, the side walls fuse, and as a result, the pieces become welded.

Understanding the mechanism of self-motion of the electric are in the channel is a separate scientific problem, and publication of a special article on this matter is planned. Here only the main reason for the phenomenon will be indicated. Moving along the channel, the arc fuses the electrode in such a way that a recession is formed in front of it. Changes in the geometry of the channel near the point of current supply to the arc bring about asymmetry of electric current lines. This, in turn, gives rise to a transverse magnetic field accelerating the arc.

Now, the thermal processes occurring in components of the assembly under this kind of welding will be considered.

Fusion of the Electrode. Since the electrode is thermally insulated from the base metal by insulating pads and the heat fluxes through its end faces y = 0 and  $y = y_0$  are comparatively low, heat propagation within the electrode is close to one-dimensional. Specifying a parabolic temperature distribution over y and integrating the three-dimensional heat conduction equation with respect to the coordinates x and y give the following mathematical model for the average temperature  $\vartheta$ :

$$\frac{\partial \vartheta}{\partial \tau} = a \frac{\partial^2 \vartheta}{\partial z^2} - \frac{2aB^y \left(\vartheta - \vartheta_0\right)}{\left(1 + B^y/6\right)y_0^2}, \quad \tau > 0, \quad f(\tau) < z < z_0;$$
(1)

$$\tau = 0: \ \vartheta = \vartheta_0, \quad f = 0, \quad z = z_0: -\lambda \frac{\partial \vartheta}{\partial z} = \alpha \left(\vartheta - \vartheta_0\right),$$
$$z = f: \ \vartheta = \vartheta_f, \quad -\lambda \frac{\partial \vartheta}{\partial z} = q_e - \rho L \nu, \tag{2}$$

describing heating and fusion of the electrode. Here  $\vartheta_0$  is the ambient temperature;  $\vartheta_j$  is the melting point of the metal; L is the latent heat of melting;  $a = \lambda/c\rho$  is the thermal diffusivity;  $B^y = \alpha y^0/\lambda$ . Stefan's boundary condition

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Fig. 1. Schematic of the assembly for welding by a fixed fusing electrode: 1) base metal; 2) metal in the weld; 3) electrode; 4) insulator; 5) arc channel; 6) arc.

(2) prescribed at the fusion front of the electrode  $f(\tau)$  contains the average heat flux density from the arc to the electrode  $q_e$  and the average welding rate  $v = df/d\tau$ . In model (1)-(2)  $q_e$  is assumed to be known. It is independent of  $\tau$ , since the scanning rate of the arc is several orders of magnitude higher than the rate of propagation of thermal disturbances in the metal. It is clear that for exact determination of  $q_e$  it is necessary to consider thoroughly the motion of the arc in the channel and the concurrent heat and mass transfer processes and subsequently to average the result. Because of the extreme complexity of the problem, we only take the very simple model in which the source density is constant:

$$q_e = P/S , (3)$$

where P is the "effective" power of the arc and S can be interpreted as the area of inner walls of the arc channel.

An analysis of problem (1)-(3) within similarity theory [3] leads to the following system of dimensionless quantities:

$$l = \lambda \frac{\vartheta_f - \vartheta_0}{q_e}, \quad Z = \frac{z - f}{l}, \quad Z_0 = \frac{z_0}{l}, \quad F = \frac{\varepsilon f}{l}, \quad \varepsilon = \frac{1}{Z_0},$$

$$t = \frac{\tau \varepsilon a}{l^2}, \quad V = \frac{\nu l}{a}, \quad T = \frac{\vartheta - \vartheta_0}{\vartheta_f - \vartheta_0}, \quad \text{Ko} = \frac{L}{c \left(\vartheta_f - \vartheta_0\right)},$$

$$B = \frac{\alpha l}{\lambda}, \quad B^{\nu} = \frac{\alpha y_0}{\lambda}, \quad D = \frac{2B^{\nu}}{1 + B^{\nu}/6} \left(\frac{l}{y_0}\right)^2,$$
(4)

substitution of which into the initial equations gives

$$\varepsilon \frac{\partial T}{\partial t} - V \frac{\partial T}{\partial Z} = \frac{\partial^2 T}{\partial Z^2} - DT, \quad t > 0, \quad 0 < Z < (1 - F)/\varepsilon,$$

$$t = 0: F = 0, \quad T = 0, \quad Z = 0: T = 1,$$

$$- \partial T / \partial Z = 1 - \text{Ko } V, \quad Z = (1 - F) / \varepsilon: - \partial T / \partial Z = BT.$$
(5)

Now, the coefficients in problem (5) will be estimated. To do this, we prescribe the characteristic thermal properties [4, 5]:  $\lambda = 30 \text{ W/(m \cdot K)}$ ,  $c = 0.5 \text{ kJ/(kg \cdot K)}$ ,  $\rho = 8 \cdot 10^3 \text{ kg/m}^3$ ,  $\vartheta_j = 1530^{\circ}\text{C}$ ,  $\theta_0 = 30^{\circ}\text{C}$ ,  $a = 7.5 \cdot 10^{-5} \text{ m}^2/\text{sec}$ , L = 300 kJ/kg,  $\alpha = 100 \text{ W/(m}^2 \cdot \text{K})$  and the geometric and operating parameters of the welding process [2]:  $y_0 = 4 \text{ cm}$ ,  $z_0 = 16 \text{ cm}$ , P = 15 kW,  $S = 4yd = 3.2 \cdot 10^4 \text{ m}^2$  (d = 2 mm is the thickness of the electrode). From formulas (3) and (4) we find  $q_e = 4.7 \cdot 10^7 \text{ W/m}^2$ , l = 0.96 mm,  $Z_0 = 160$ ,  $\varepsilon = 6.2 \cdot 10^{-3}$ , Ko = 0.4,  $B = 3.3 \cdot 10^{-3}$ ,  $D = 10^{-4}$ . The quantity  $l \approx 1 \text{ mm}$  is actually the thickness of the thermal boundary layer before the melting front. The smallness of the ratio of  $\varepsilon$  to the parameters B and D indicates a slight effect of the boundary conditions on the behavior of the temperature at a long distance from the boundaries of the electrode. In the first approximation, at  $\varepsilon = 0$ , D = 0, and B = 0, the solution of problem (5) has the form

$$V = (1 + Ko)^{-1}, \quad T = \exp(-VZ).$$
 (6)

It describes adequately the heating and fusion of the electrode everywhere except the boundary regions, i.e., when  $F < Z_0$  or, with account for the equality F = Vt, when  $t < (1 + K_0)/\varepsilon$ . In terms of dimensional quantities the range of validity of Eq. (6) can be expressed by the formula  $l < z_0 - f$ .

The solution obtained shows that in the welding method described the electrode fuses at the constant rate  $v = Va/l \approx 1$  cm/sec and the fusion front is ahead of thermal disurbances, and therefore the electrode remains relatively cold, except for a thin boundary layer with the thickness  $l \approx 1$  mm.

Heating of the Base Metal. In mathematical modeling of the behavior of the temperature field in the base metal, use is made of the fact that the cross-sectional dimension d of the channel is much smaller than the characteristic dimension of the piece, in particular, the thickness  $X_0$ . In this case a real heat source can be replaced by a source with equivalent power, operating along the line  $z = f(\tau)$  in the plane x = 0. Moreover, it seems impracticable to consider fusion of the channel walls since the heat spent on fusion is released immediately in crystallization. Another distinctive feature of the source model is inclusion of transfer of melt drops from the electrode to the other walls of the channel, followed by crystallization. As a result, an additional amount of heat proportional to the welding rate is released:

$$q_m = q_e + \rho L v d / l_m \,. \tag{7}$$

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Here the subscript *m* indicates the other three walls of the arc channel formed by the base metal and the weld. For example, for a square channel with the side *d*,  $l_m = 3d$ . In view of the above said, the heat source model can be represented in the form of a boundary condition for the problem on heating of the base metal:

$$x = 0; \quad -\lambda \frac{\partial \vartheta}{\partial x} = q_* \delta \left( z - f \right), \tag{8}$$

where  $q_* = q_m = q_m l_m/2$ ,  $\delta(\xi)$  is the Dirac function. Just as in the problem on fusion of the electrode, the initial formulation of the problem can be simplified, having specified a parabolic temperature distribution over y. In this case, integration gives a two-dimensional equation similar to Eq. (1). In an analysis of the corresponding initial-value problem, it is necessary to bear in mind that the scales of time and length are dictated by solution (6) and are equal, respectively, to

$$\tau_0 = z_0 / \nu = l_0 z_0 / \nu a , \quad l_0 = \sqrt{a \tau_0} .$$
 (9)

In this case the dimensionless variables are determined by the relations

$$t = \tau/\tau_0, \quad X = x/l_0, \quad Z = z/l_0, \quad B_0 = \alpha l_0/\lambda,$$
  

$$T = (\vartheta - \vartheta_0)/(\vartheta_f - \vartheta_0), \quad Q = q_*/\lambda (\vartheta_f - \vartheta_0),$$
(10)



Fig. 2. Dynamics of the temperature field in the base metal.

and the mathematical model of the average temperature T(X, Z, t) over y has the form

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Z^2} - DT, \quad t < 0 < 1, \quad 0 < Z < Z_0,$$
(11)

$$t = 0: T = 0, Z = 0, Z = Z_0, X = X_0: \partial T / \partial n = -B_0 T,$$
 (12)

$$X = 0: \ \partial T / \partial X = -B\delta \left( Z - Vt \right). \tag{13}$$

According to formulas (7)-(10), typical values of parameters in problem (11)-(13) are as follows:  $\tau_0 = 16$  sec,  $l_0 = 1.1$  cm,  $Z_0 = 14.5$ ,  $B_0 = 0.037$ , Q = 3.53, and  $D = 10^{-4}$ . Just as in problem (5), thermal interaction with the environment is negligible (D << 1,  $B_0 << 1$ ). Since  $X_0 > 1$ , during the welding the heat wave from the source does not have enough time to reach the boundary of the body  $X = X_0$ . Because of this and also since  $Z_0 >> 1$ , it is possible to state that the solution of problem (11)-(13), at least at a long distance from the boundaries, is close to the function

$$\widetilde{T}(X, Z, t) = \frac{Q}{2\Pi} \int_{0}^{t} \exp\left\{\frac{\left[Z - V(t - \tau)\right]^{2} + X^{2}}{4\tau}\right\} \frac{d\tau}{\tau},$$
(14)

describing propagation of heat from a moving source in a semi-infinite body [6].

In the numerical simulation we used the method of finite elements, and the discrete model was based on an integral identity determining the generalized solution of problem (11)-(13) [7]. Linear triangular elements were used; for approximation of a moving point source the quantity Q in boundary condition (13) was "smeared" over two neighboring nodes of the grid between which the source occurred at the given moment. On each time layer  $t_j$ , a system of finite-element equations was solved by the iteration method of conjugate directions [8].

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Fig. 3. Isotherms in the developed phase of welding: a) numerical simulation; b) calculated by formula (14).

Results of the numerical experiment fully confirm theoretical concepts of the dynamics of heating of the base metal in welding by a fixed fusing electrode. Figure 2 illustrates the propagation of thermal waves from a moving source to "cold" metal (isotherms are shown in the range 0 < T < 1 with the step 0.05). Special calculations have shown that changes in the coefficients D and  $B_0$  have almost no effect on the solution of the problem and can be assumed to be zero. Moreover, in view of the fact that T is directly proportional to Q and V = 1/Z, it can be concluded that important parameters determining the dynamics of the temperature field in the base metal are  $X_0$  and  $Z_0$ , the former of which ( $X_0 > 1$ ) also does not have any effect on the solution in real situations.

In Fig. 3 the numerical solution of the problem (a) is compared with the temperature field (b) calculated by formula (14) in the developed phase of welding at t = 0.6. It can be easily seen that all important differences are concentrated in regions adjacent to the boundaries of the body.

Formation of Defects in the Weld. The insulating pads that separate the fusing electrode from the base material burn out in welding. This leads to a deficit of the metal volume in the weld and to an increase in the gap between the upper and lower walls of the arc channel. From the conservation of mass (changes in the density are neglected) the following relation is obtained:

$$\Delta(\tau) = \Delta(0) + 2h/d\nu\tau, \qquad (15)$$

which relates the increase in the gap with the thickness of the insulator h, the thickness of the electrode d, and the rate of welding v. According to Eq. (15), at h = 0.1 mm, v = 1 cm/sec, and d = 2 mm, the gap can be 10% of the length of the piece. However, when the gap increases substantially, combustion of the arc between the electrode and the lower wall of the channel becomes impossible and the arc occupies a position more favorable as regards power, at a longer distance from the electrode and closer to the side wall. In this case drops of the melt transferred by the arc are crystallized on the side surfaces, which provides favorable conditions for forming gas pores and compensation of the volume deficit because of them. This hyposesis of the reason for defects in the weld produced by this method of welding can be verified by measuring the electric current field in the base metal. The point is that because of the strong dependence of the conductivity of the metal  $\sigma$  on the temperature T and nonuniform heating of the piece, the electric current lines depend substantially on the direction of combustion of the arc. For illustration, the model problem of an electric field in the plane XOZ will be considered. In dimensionless variables the equation for the potential U has the form

div 
$$\sigma(T)$$
 grad  $U = 0$ ,  $Z = 0$ :  $U = 0$ ;



Fig. 4. Asymmetry of the electric current lines with arc burning on the side wall of the channel.

$$\partial U/\partial n = \delta (X - X_*, Z - Z_*)$$
 on the boundary. (10)

The point of the boundary  $(X_*, Z_*)$  at which the arc burns can be located both on the lower and on the side wall of the arc channel. In the first approximation the relation  $\sigma(T)$  can be approximated by the formula

$$\sigma = \left(1 + \beta \widetilde{T}\right)^{-1},\tag{17}$$

where the temperature coefficient  $\beta \approx 7$  [5]. In the solution of problem (16) use was made of the temperature distribution prescribed by Eq. (14). Results of the numerical simulation have shown that with normal burning of the arc the current field is symmetric. Two cases were considered for lateral orientation of the arc. In the first case the dependence  $\sigma(T)$  was neglected and asymmetry of the field was observed only in the neighborhood of the source. In the second case the real situation, when the conductivity is determined by Eq. (17), was simulated. The pattern of the electric current lines changes fundamentally (see Fig. 4). Substantial heating of the weld line prevents the field from leveling and in one half of the welded piece the current density turns out to be approximately twice as high as in the other.

**Conclusion.** Theoretical and numerical analyses of the processes occurring in welding by a fixed fusing electrode allow the following conclusions.

The welding rate is constant, independent of the size of pieces to be welded, and is determined only by the size of the electrode and the current power. Due to the high rate of welding, the temperature field is nonuniform: the line of the weld is heated substantially, while the peripheral regions of the piece remain cold throughout the welding. Consequently, the environmental effects on the temperature field are insignificant: they are pronounced only in cooling of the piece. The deficit of metal in the weld caused by burn-out of the insulator leads inevitably to an increase in the arc gap and induces formation of pores, which narrows the region of possible application of this welding method.

## NOTATION

Dimensional quantities: x, y, z, coordinates;  $\tau$ , time;  $x_0$ ,  $y_0$ ,  $z_0$ , boundaries of the body; d, thickness of the electrode;  $\Delta$ , gap between the electrode and the weld metal;  $\theta$ , temperature;  $\vartheta_0$ , ambient temperature;  $\vartheta_j$ , melting point;  $\lambda$ , thermal conductivity; c, heat capacity;  $\rho$ , density; a, thermal diffusivity;  $\alpha$ , heat transfer coefficient; L,

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heat of melting; f, front of fusion of the electrode; v, rate of welding; l,  $l_0$ ,  $l_m$ , scales of length. Dimensionless quantities: X, Y, Z, coordinates; t, time;  $X_0$ ,  $Z_0$ , boundaries of the body; T, temperature, F, front of fusion of the electrode; V, rate of welding; Q, heat flux density; B,  $B^y$ , Biot numbers; Ko, Kossovich criterion; D, intensity of heat removal; U, potential of the electric field;  $\beta$ , temperature coefficient in the relation  $\sigma(T)$ .

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